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Solutions of Luikov equations of heat and mass transfer in capillary-porous bodies

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Abstract—This paper presents an analytical method to solve the Luikov system of linear partial differential equations subject to specified initial and boundary conditions. Luikov equations are the governing equations in analyzing heat and mass diffusion problems for capillary-porous bodies. However, an analytical method to obtain complete and satisfactory solutions of these equations is still lacking in the literature. The method of solution presented in this paper is illustrated by considering the transient distributions of temperature and moisture in a slab of wood during drying. Numerical results are obtained and compared with published finite element solutions and experimental data for spruce specimens. The method should have a general application to problems of heat and mass transfer in capillary-porous bodies.

INTRODUCTION

THIS PAPER presents a method of solution for the system of linear partial differential equations derived by Luikov [1]. The Luikov system of equations is a non-linear system because the transfer coefficients are functions of either moisture content or temperature. To make this system more mathematically tractable, Luikov and Mikhailov [2] suggested that calculations of time-dependent heat and mass transfer be made assuming constant transfer coefficient zones (average values can be taken for each zone). Therefore, an efficient method of solution for the linear system with constant transfer coefficients is instrumental in solving the non-linear system of equations. For the simple cases of a slab, a cylinder, and a sphere, Luikov and Mikhailov [2] used the Laplace transform technique to obtain their solutions. These same problems were also treated by Mikhailov and Özişik [3] using the finite integral transform technique. They obtained the same solutions as those of Luikov and Mikhailov [2]. However, as Lobo *et al.* [4] recently indicated, these solutions ignored the possible existence of complex eigenvalues, and the solutions must therefore be re-examined. If complex eigenvalues do exist, these solutions can be grossly in error.

In our study, we developed an analytical approach that yields complete and satisfactory solutions for the Luikov equations subject to specified initial and boundary conditions. The temperature and moisture potential are expressed in terms of a potential function reducing the two coupled Luikov equations to a single fourth-order partial differential equation, with the potential function as the dependent variable. The solution of this equation leads to separate expressions for the temperature and moisture potential, which are coupled by the boundary conditions. From the boundary conditions we obtain a transcendental equation that can be satisfied by an infinite number of real eigenvalues. Using the method of Müller [5], we found that the equation also has a pair of complex roots. Thus, the temperature and moisture potential are expressed in terms of the eigenvalues and an equal number of unknown coefficients in the form of infinite series. Making use of the initial conditions, the unknown coefficients may be evaluated using a least-squares technique [6, 7]. When the complex eigenvalues are not included in the infinite series, the solutions cannot satisfy the initial conditions. We show that both real and complex eigenvalues are needed to yield results that can satisfy all the conditions of the problem.

The significance of the present study is threefold: (1) our method of solution should have a general application to problems of heat and mass transfer in capillary-porous bodies; (2) our solutions provide an approach to evaluate the relative importance of the thermophysical properties of capillary-porous materials in a heat and mass transfer analysis; and (3) our solutions serve to gauge the accuracy of any numerical approaches such as the finite element and the finite difference techniques when applied to solve this type of problem.

HEAT AND MASS TRANSFER EQUATIONS

For the one-dimensional case as shown in Fig. 1, heat and moisture move along the x-axis only. Under

NOMENCLATURE

A, B	arbitrary constant coefficients	Greek symbols	
C	moisture content	α_m	convective mass transfer coefficient
C_m	moisture capacity	α_q	convective heat transfer coefficient
C_q	heat capacity	δ	thermographic coefficient
g	B/A	ε	ratio of vapor diffusion coefficient to coefficient of total moisture diffusion
K_m	moisture conductivity coefficient		
K_q	thermal conductivity coefficient	λ	heat of phase change
l	half thickness of specimen	ρ	dry body density
T_a	temperature of drying medium	ϕ	potential function.
T_0	initial temperature		
t	time	Subscripts	
U	moisture potential	a	surrounding medium
U_a	moisture potential of drying medium	m	mass transfer
		q	heat transfer
U_0	initial moisture potential.	0	initial condition.

constant pressure condition, Luikov [1] equations can be written as follows:

$$\rho C_q \frac{\partial T}{\partial t} = K_q \frac{\partial^2 T}{\partial x^2} + \varepsilon \lambda \rho C_m \frac{\partial U}{\partial t} \tag{1}$$

and

$$\rho C_m \frac{\partial U}{\partial t} = K_m \delta \frac{\partial^2 T}{\partial x^2} + K_m \frac{\partial^2 U}{\partial x^2} \tag{2}$$

where T is the temperature, U the moisture potential, t the time, K_q and K_m the thermal and moisture conductivity coefficients, respectively, C_q and C_m the heat and moisture capacities, respectively, ρ the dry body density, ε the ratio of the vapor diffusion coefficient to the coefficient of total moisture diffusion, λ the heat of phase change, and δ the thermogradient coefficient. The moisture potential U is related to the moisture content C by

$$C = C_m U. \tag{3}$$

At the surfaces of the specimen in Fig. 1, $x = \pm l$, the boundary conditions of the third kind [1] apply. They are

$$K_q \frac{\partial T}{\partial x} + \alpha_q (T - T_a) + (1 - \varepsilon)\lambda \alpha_m (U - U_a) = 0 \tag{4}$$

and

$$K_m \frac{\partial U}{\partial x} + K_m \delta \frac{\partial T}{\partial x} + \alpha_m (U - U_a) = 0 \tag{5}$$

where α_q and α_m are convective heat and mass transfer coefficients, respectively, and T_a and U_a the temperature and moisture potential of the drying medium.

Because of symmetry, at $x = 0$ we should have

$$\frac{\partial T}{\partial x} = 0 \tag{6}$$

and

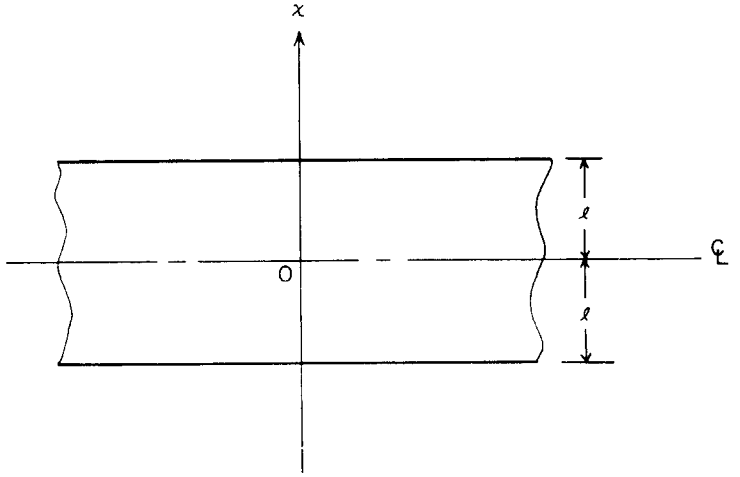

FIG. 1. Schematic representation of wood specimen.

$$\frac{\partial U}{\partial x} = 0. \quad (7)$$

The initial conditions are assumed to be constant and are represented by

$$T(x, 0) = T_0 \quad (8)$$

and

$$U(x, 0) = U_0. \quad (9)$$

METHOD OF SOLUTION

Introducing two functions T_1 and U_1 of x and t such that

$$T(x, t) = T_1(x, t) + T_a \quad (10)$$

$$U(x, t) = U_1(x, t) + U_a \quad (11)$$

and substituting T and U in all the preceding equations, we obtain the following.

1. *Basic equations*

$$\rho C_q \frac{\partial T_1}{\partial t} = K_q \frac{\partial^2 T_1}{\partial x^2} + \varepsilon\lambda\rho C_m \frac{\partial U_1}{\partial t} \quad (12)$$

$$\rho C_m \frac{\partial U_1}{\partial t} = K_m \delta \frac{\partial^2 T_1}{\partial x^2} + K_m \frac{\partial^2 U_1}{\partial x^2}. \quad (13)$$

2. *Boundary conditions*

$$K_q \frac{\partial T_1}{\partial x} + \alpha_q T_1 + (1-\varepsilon)\lambda\alpha_m U_1 = 0 \quad (x = l) \quad (14)$$

$$K_m \frac{\partial U_1}{\partial x} + K_m \delta \frac{\partial T_1}{\partial x} + \alpha_m U_1 = 0 \quad (x = l) \quad (15)$$

$$\frac{\partial T_1}{\partial x} = 0 \quad (x = 0) \quad (16)$$

$$\frac{\partial U_1}{\partial x} = 0 \quad (x = 0). \quad (17)$$

3. *Initial conditions*

$$T_1(x, 0) = T_0 - T_a \quad (18)$$

$$U_1(x, 0) = U_0 - U_a. \quad (19)$$

Introducing a potential function $\phi(x, t)$ such that

$$T_1 = \left(K_m \frac{\partial^2}{\partial x^2} - \rho C_m \frac{\partial}{\partial t}\right)\phi \quad (20)$$

and

$$U_1 = -K_m \delta \frac{\partial^2 \phi}{\partial x^2} \quad (21)$$

we find that equation (13) is automatically satisfied and equation (12) becomes

$$\left[\frac{\partial^4}{\partial x^4} - \frac{\rho}{K_m K_q}(C_m K_q + C_q K_m + \varepsilon\lambda C_m K_m \delta)\frac{\partial^3}{\partial x^2 \partial t} + \frac{\rho^2 C_m C_q}{K_m K_q}\frac{\partial^2}{\partial t^2}\right]\phi = 0. \quad (22)$$

Let

$$2C_1 = \frac{\rho}{K_m K_q}(C_m K_q + C_q K_m + \varepsilon\lambda C_m K_m \delta) \quad (23)$$

and

$$C_2^2 = \frac{\rho^2}{K_m K_q} C_m C_q. \quad (24)$$

Equation (22) can be written as

$$\left(\frac{\partial^4}{\partial x^4} - 2C_1 \frac{\partial^2}{\partial x^2}\frac{\partial}{\partial t} + C_2^2 \frac{\partial^2}{\partial t^2}\right)\phi = 0$$

or

$$\left(\frac{\partial^2}{\partial x^2} - C_1 \frac{\partial}{\partial t} - \sqrt{(C_1^2 - C_2^2)}\frac{\partial}{\partial t}\right) \times \left(\frac{\partial^2}{\partial x^2} - C_1 \frac{\partial}{\partial t} + \sqrt{(C_1^2 - C_2^2)}\frac{\partial}{\partial t}\right)\phi = 0 \quad (25)$$

which can be broken into two equations as follows:

$$\frac{\partial^2 \phi_1}{\partial x^2} = D_1^2 \frac{\partial \phi_1}{\partial t} \quad (26)$$

$$\frac{\partial^2 \phi_2}{\partial x^2} = D_2^2 \frac{\partial \phi_2}{\partial t} \quad (27)$$

with

$$D_1^2 = C_1 + \sqrt{(C_1^2 - C_2^2)} \quad (28)$$

and

$$D_2^2 = C_1 - \sqrt{(C_1^2 - C_2^2)}. \quad (29)$$

Equations (26) and (27) are of the diffusion type. The general solution of equation (25) is the sum of the solutions of equations (26) and (27) and can therefore be written as

$$\phi = \phi_1(x, t) + \phi_2(x, t)$$

in which

$$\phi_1(x, t) = A\,e^{-\xi^2 t} \cos D_1 \xi x$$

$$\phi_2(x, t) = B\,e^{-\xi^2 t} \cos D_2 \xi x.$$

Hence

$$\phi = e^{-\xi^2 t}(A \cos D_1 \xi x + B \cos D_2 \xi x) \quad (30)$$

where A and B are arbitrary constant coefficients. (Sine functions do not appear because of the conditions of symmetry represented by equations (16) and (17).)

Substituting equation (30) into equations (20) and (21) yields

$$T_1 = \xi^2 e^{-\xi^2 t}[(-K_m D_1^2+\rho C_m)A \cos D_1\xi x + (-K_m D_2^2+\rho C_m)B \cos D_2\xi x] \quad (31)$$

$$U_1 = K_m\delta\xi^2 e^{-\xi^2 t}(D_1^2 A \cos D_1\xi x + D_2^2 B \cos D_2\xi x). \quad (32)$$

From these expressions and the boundary conditions (14) and (15), we obtain

$$(a_1\xi \sin D_1\xi l + a_2 \cos D_1\xi l)A + (a_3\xi \sin D_2\xi l + a_4 \cos D_2\xi l)B = 0 \quad (33)$$

$$(b_1\xi \sin D_1\xi l + b_2 \cos D_1\xi l)A + (b_3\xi \sin D_2\xi l + b_4 \cos D_2\xi l)B = 0 \quad (34)$$

where

$$a_1 = K_q D_1(K_m D_1^2 - \rho C_m)$$
$$a_2 = K_m D_1^2[(1-\varepsilon)\lambda\alpha_m\delta - \alpha_q] + \alpha_q\rho C_m$$
$$a_3 = K_q D_2(K_m D_2^2 - \rho C_m)$$
$$a_4 = K_m D_2^2[(1-\varepsilon)\lambda\alpha_m\delta - \alpha_q] + \alpha_q\rho C_m$$
$$b_1 = -\rho C_m D_1$$
$$b_2 = \alpha_m D_1^2$$
$$b_3 = -\rho C_m D_2$$
$$b_4 = \alpha_m D_2^2.$$

For non-trivial solutions of equations (33) and (34) to exist, the determinant of the coefficients of A and B must vanish, giving the transcendental equation

$$(D_1\xi \tan D_1\xi l + \psi_1)(D_2\xi \tan D_2\xi l + \psi_2) = \psi_3 \quad (35)$$

in which

$$\psi_1 = \frac{\alpha_q K_m D_1^2 - \alpha_q\rho C_m - (1-\varepsilon)\lambda\alpha_m K_m\delta D_1^2 + \alpha_m K_q D_1^2(1 - K_m D_2^2/\rho C_m)}{K_q K_m(D_2^2 - D_1^2)}$$

$$\psi_2 = \frac{-\alpha_q K_m D_2^2 - \alpha_q\rho C_m - (1-\varepsilon)\lambda\alpha_m K_m\delta D_2^2 + \alpha_m K_q D_2^2(1 - K_m D_1^2/\rho C_m)}{K_q K_m(D_2^2 - D_1^2)}$$

$$\psi_3 = -\frac{\alpha_m\alpha_q}{K_m K_q} + \psi_1\psi_2.$$

From equation (34) we can also derive

$$\frac{B}{A} = \frac{-\rho C_m D_1\xi \sin D_1\xi l + \alpha_m D_1^2 \cos D_1\xi l}{\rho C_m D_2\xi \sin D_2\xi l - \alpha_m D_2^2 \cos D_2\xi l} = g(\xi) \quad (36)$$

where the ratio B/A is set equal to g so that B can be expressed in terms of A or A in terms of B.

In equation (35) the constant parameter ξ can take an infinite number of real values and can also take some complex values. These values are called eigenvalues of ξ. For each eigenvalue, corresponding values for A and B should exist. Therefore, equations (31) and (32) can be put in the following series form:

$$T_1 = \sum_{n=1}^{\infty} \xi_n^2 e^{-\xi_n^2 t} A_n[(-K_m D_1^2 + \rho C_m) \times \cos D_1\xi_n x + (-K_m D_2^2 + \rho C_m)g(\xi_n) \times \cos D_2\xi_n x] \quad (37)$$

and

$$U_1 = \sum_{n=1}^{\infty} K_m\delta\xi_n^2 e^{-\xi_n^2 t} A_n[D_1^2 \cos D_1\xi_n x + D_2^2 g(\xi_n) \cos D_2\xi_n x] \quad (38)$$

where the function $g(\xi_n)$ from equation (36) is used to eliminate B_n. Since ξ_n can be either positive or negative in the preceding equations without changing the results, we need to take only positive real values and complex values with positive real components in the following calculations.

Now we must evaluate the coefficient A_n in equations (37) and (38). These coefficients are independent of time. By setting $t = 0$ in equations (37) and (38) and making use of the initial conditions in equations (18) and (19), these coefficients can be evaluated using a least-squares technique [6, 7]. First, we set up the following integral:

$$\Omega = \int_0^l \Bigg\{\left[\sum_{n=1}^{\infty} \xi_n^2(a_5 \cos D_1\xi_n x + a_6 g(\xi_n) \cos D_2\xi_n x)A_n - a_7\right] \times \left[\sum_{n=1}^{\infty} \bar{\xi}_n^2(a_5 \cos D_1\bar{\xi}_n x + a_6 g(\bar{\xi}_n) \cos D_2\bar{\xi}_n x)\bar{A}_n - a_7\right] + \left[\sum_{n=1}^{\infty} \xi_n^2(b_5 \cos D_1\xi_n x + b_6 g(\xi_n) \cos D_2\xi_n x)A_n - b_7\right] \times \left[\sum_{n=1}^{\infty} \bar{\xi}_n^2(b_5 \cos D_1\bar{\xi}_n x + b_6 g(\bar{\xi}_n) \cos D_2\bar{\xi}_n x)\bar{A}_n - b_7\right]\Bigg\} dx \quad (39)$$

which must be a minimum and in which

$$a_5 = -K_m D_1^2 + \rho C_m$$
$$a_6 = -K_m D_2^2 + \rho C_m$$
$$a_7 = T_0 - T_a$$
$$b_5 = K_m\delta D_1^2$$

$$b_6 = K_m \delta D_2^2$$

$$b_7 = U_0 - U_a.$$

The parameters $\bar{\xi}_n$ and $\bar{A}_n$ are complex conjugates of ξ_n and A_n, respectively.

The condition that Ω be a minimum requires that its partial derivatives with respect to $\bar{A}_m$ or A_m shall be zero. We therefore have

$$\frac{\partial \Omega}{\partial \bar{A}_m} = \int_0^l \Bigg\{ \Bigg[\sum_{n=1}^{\infty} \xi_n^2 (a_5 \cos D_1 \xi_n x + a_6 g(\xi_n) \cos D_2 \xi_n x) A_n - a_7 \Bigg] \times \bar{\xi}_m^2 (a_5 \cos D_1 \bar{\xi}_m x + a_6 g(\bar{\xi}_m) \cos D_2 \bar{\xi}_m x) + \Bigg[\sum_{n=1}^{\infty} \xi_n^2 (b_5 \cos D_1 \xi_n x + b_6 g(\xi_n) \cos D_2 \xi_n x) A_n - b_7 \Bigg] \times \bar{\xi}_m^2 (b_5 \cos D_1 \bar{\xi}_m x + b_6 g(\bar{\xi}_m) \cos D_2 \bar{\xi}_m x) \Bigg\} \mathrm{d}x = 0 \quad (m = 1, 2, 3, \ldots). \tag{40}$$

Note that the same results are obtained if we set $\partial \Omega / \partial A_m = 0$. In matrix form, equation (40) generates a Hermitian matrix as

$$[C_{mn}]\{A_n\} = \{R_m\} \tag{41}$$

in which

$$C_{mn} = \int_0^l \Big\{ [\bar{\xi}_m^2 (a_5 \cos D_1 \bar{\xi}_m x + a_6 g(\bar{\xi}_m) \cos D_2 \bar{\xi}_m x) \xi_n^2 (a_5 \cos D_1 \xi_n x + a_6 g(\xi_n) \cos D_2 \xi_n x)] + [\bar{\xi}_m^2 (b_5 \cos D_1 \bar{\xi}_m x + b_6 g(\bar{\xi}_m) \cos D_2 \bar{\xi}_m x) \xi_n^2 (b_5 \cos D_1 \xi_n x + b_6 g(\xi_n) \cos D_2 \xi_n x)] \Big\} \mathrm{d}x \tag{42}$$

and

$$R_m = \int_0^l \bar{\xi}_m^2 [(a_7 a_5 + b_7 b_5) \cos D_1 \bar{\xi}_m x + (a_7 a_6 + b_7 b_6) g(\bar{\xi}_m) \cos D_2 \bar{\xi}_m x] \, \mathrm{d}x. \tag{43}$$

The coefficients A_n can be determined from the system of linear equations (41). We can then calculate T_1 and U_1 from equations (37) and (38), and T and U from equations (10) and (11).

NUMERICAL EXAMPLE AND DISCUSSION

The thermophysical properties of spruce and other input data used by Thomas *et al.* [8] will be used in the numerical calculations so that the results can be directly compared with the finite element predictions of these authors. Since the purpose of the study by Thomas *et al.* [8] was to check the experimental data of Keylwerth [10], we will also include Keylwerth's data in our comparison. The input data are as follows:

thermal conductivity coefficient, $K_q = 0.65$ W m^{-1} K^{-1}

moisture conductivity coefficient, $K_m = 2.2 \times 10^{-8}$ kg m^{-1} s^{-1} °M^{-1}

heat capacity, $C_q = 2500$ J kg^{-1} K^{-1}

moisture capacity, $C_m = 0.01$ kg (moisture) (kg (dry body))$^{-1}$ °M^{-1}

dry body density, $\rho = 370$ kg m^{-3}

ratio of vapor diffusion coefficient to coefficient of total moisture diffusion, $\varepsilon = 0.3$

heat of phase change (from ref. [9]), $\lambda = 2.5 \times 10^6$ J kg^{-1}

thermogradient coefficient, $\delta = 2.0$°M K^{-1}

convective heat transfer coefficient, $\alpha_q = 22.5$ W m^{-2} K^{-1}

convective mass transfer coefficient, $\alpha_m = 2.5 \times 10^{-6}$ kg m^{-2} s^{-1} °M^{-1}

half thickness of specimen in radial direction, $l = 0.012$ m

initial temperature, $T_0 = 10$°C

air temperature, $T_a = 110$°C

initial moisture potential, $U_0 = 86$°M

air moisture potential, $U_a = 4$°M.

We note that the constant thermophysical properties listed are only for numerical comparison with existing data in the literature. In reality, these properties may be functions of either moisture content or temperature, requiring the application of zonal calculations [2] for improved solutions. We can also apply numerical techniques such as the finite element method used by Thomas *et al.* [9] and the finite difference approach employed by Stanish *et al.* [11] to account for variable thermophysical properties.

The real eigenvalues in equation (35) were obtained using a bisection procedure. For the complex roots, we used a method by Müller [5]. After an exhaustive search for all the complex roots, we could find only one pair. They are as follows:

$$4.15949 \times 10^{-2} \pm 1.61002 \times 10^{-3}\mathrm{i}.$$

The system of linear equations (41) converges rapidly. As pointed out previously, the complex roots were overlooked in all the previous studies in the literature that used analytical methods. To examine the effect of the pair of complex roots on the final solutions, we checked the initial conditions at the specimen center by first considering real eigenvalues only and then including the complex eigenvalues. The

Table 1. Temperature and moisture potential for the first n real eigenvalues[a]

	n					
	10	20	30	40	50	60
T_0 (K)	339.8	345.8	344.0	345.0	344.5	344.5
U_0 (°M)	80.4	81.2	81.0	81.1	81.1	81.1

[a] $x = 0$, $t = 0$.

results obtained from the first n real roots for T_0 and U_0 are shown in Table 1.

The input data are $T_0 = 283.15$ K or 10°C and $U_0 = 86$°M. By increasing the number of terms without considering the complex roots, we clearly cannot satisfy the initial conditions (see Table 1). When we used the first 38 real roots plus the above pair of complex roots, we obtained $T_0 = 283.4$ K and $U_0 = 85.99$°M, which are essentially the same as the input values.

Figure 2 shows the variations of temperature at the specimen center and surface as a function of time. For the initial period of less than 10 min, the complex eigenvalues had a very significant effect.

The variations of moisture content at the specimen center and surface as a function of time are presented in Fig. 3. As shown, the moisture content at the surface may be higher than that at the center when the complex eigenvalues are not included in the numerical calculations. This is, of course, impossible in a drying environment. Therefore, excluding the complex eigenvalues in the analysis results in a solution that is not only incomplete but also erroneous.

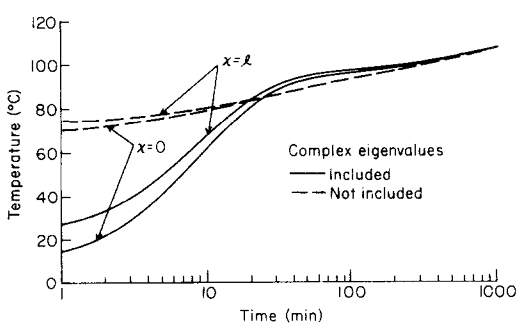

FIG. 2. Temperature at center and surface of spruce specimen.

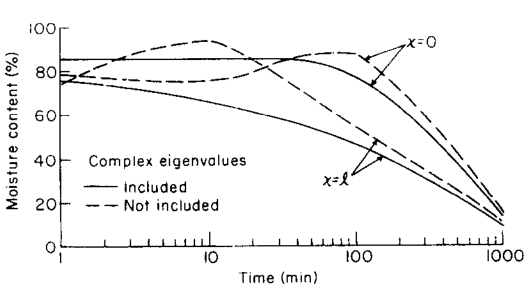

FIG. 3. Moisture content at center and surface of spruce specimen.

Figure 4 presents the finite element results by Thomas *et al.* [8], the experimental data by Keylwerth [10], and the solution of the present work for the variation in surface temperature with time. Results from the same sources for variation in center temperature with time are plotted in Fig. 5. The discrepancies between the analytical results and the experimental data can be explained by the constant thermophysical properties of spruce used in the calculations and the difficulties in experimental measurements. The finite element solutions closely follow the general trend of the analytical results. The accuracy of the finite element solutions depends on, among other things, the refinement of the elements as well as the criterion of convergence, which are not disclosed in Thomas *et al.* [8].

The moisture variations at the surface and center of the specimen with time are shown in Figs. 6 and 7, respectively. As discussed by Thomas *et al.* [8], the crooked shape of the experimental curve between $t = 30$ and 200 min in Fig. 6 was not explained by Keylwerth [10] and was therefore considered to be suspect. Thomas *et al.* [8] also mentioned that their finite element analysis was modeled in two stages so that their results would agree with experimental data up to $t = 10$ min. They did not give the details of this two-stage modeling. No such adjustments were made in the present work.

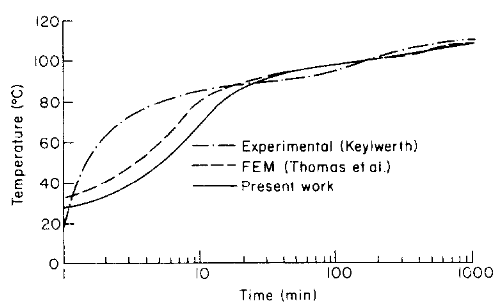

FIG. 4. Temperature at surface of spruce specimen. Lines represent experimental data of Keylwerth [10], finite element method (FEM) of Thomas *et al.* [8], and results of work reported here.

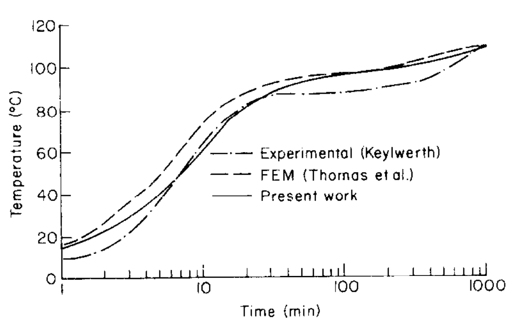

FIG. 5. Temperature at center of spruce specimen. Lines represent experimental data of Keylwerth [10], finite element method (FEM) of Thomas *et al.* [8], and results of work reported here.

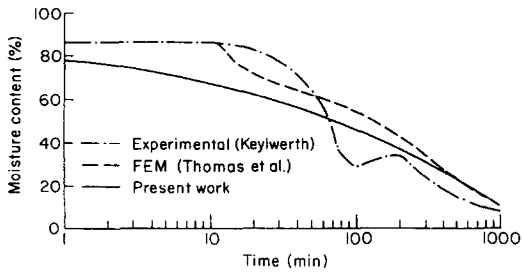

FIG. 6. Moisture content at surface of spruce specimen. Lines represent experimental data of Keylwerth [10], finite element method (FEM) of Thomas *et al.* [8], and results of work reported here.

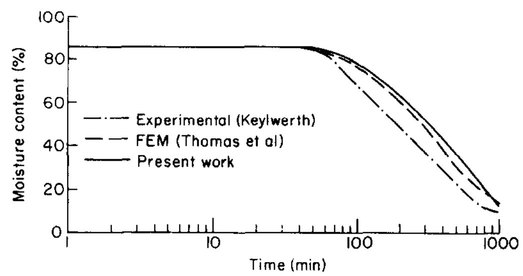

FIG. 7. Moisture content at center of spruce specimen. Lines represent experimental data of Keylwerth [10], finite element method (FEM) of Thomas *et al.* [8], and results of work reported here.

CONCLUSIONS

This paper presents an analytical method for solving the Luikov equations for heat and mass transfer to predict the temperature and moisture distributions in capillary-porous bodies during drying. The inclusion of the complex eigenvalues in the analysis was found to be of substantial importance. Without these values, as was the case in the other known analyses in the literature, the solutions are not only incomplete but physically unreasonable. Numerical results compare reasonably well with published experimental data for spruce specimens and can serve to evaluate the accuracy of any approximate numerical methods, such as finite element and finite difference techniques, when applied to solve this kind of problem.

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SOLUTIONS DES EQUATIONS DE LUIKOV POUR LE TRANSFERT DE CHALEUR ET DE MASSE DANS LES CORPS A POROSITE CAPILLAIRE

Résumé—On présente une méthode analytique pour résoudre le système d'équations linéaires aux dérivées partielles de Luikov avec des conditions spécifiées initiales et aux limites. Ces équations de Luikov sont celles qui régissent les problèmes de diffusion de chaleur et de masse pour les corps à porosité capillaire. Néanmoins une méthode analytique est absente dans la littérature. La méthode présentée ici est illustrée en considérant les distributions de température et d'humidité dans une couche de bois pendant le séchage. Des résultats numériques sont obtenus et comparés avec des solutions aux éléments finies déjà publiées et des données expérimentales sur des spécimens en spruce. La méthode pourrait avoir une application générale aux problèmes dans les corps à porosité capillaire.

LÖSUNG DER LUIKOV-GLEICHUNG FÜR DIE WÄRME- UND STOFFÜBERTRAGUNG IN KAPILLARPORÖSEN KÖRPERN

Zusammenfassung—In der vorliegenden Arbeit wird ein analytisches Verfahren zur Lösung des Systems linearer partieller Differentialgleichungen nach Luikov bei gegebenen Anfangs- und Randbedingungen vorgeschlagen. Als Luikov-Gleichungen werden die maßgeblichen Gleichungen für Probleme des Wärme- und Stofftransports in kapillarporösen Körpern bezeichnet. Bisher wird in der Literatur kein befriedigendes vollständiges analytisches Lösungsverfahren für diese Gleichungen beschrieben. Das hier vorgestellte Lösungsverfahren wird anhand der zeitlichen Verteilung von Temperatur und Feuchtigkeit während des Trocknungsvorgangs in einer Holzplatte verdeutlicht. Es werden numerische Lösungen ermittelt und mit veröffentlichten Finite-Elemente-Lösungen sowie mit experimentellen Daten für Fichtenholzproben verglichen. Das vorgestellte Verfahren sollte allgemein auf Probleme der Wärme- und Stoffübertragung in kapillarporösen Körpern anwendbar sein.

РЕШЕНИЕ УРАВНЕНИЙ ЛЫКОВА, ОПИСЫВАЮЩИХ ТЕПЛО- И МАССОПЕРЕНОС В КАПИЛЛЯРНО-ПОРИСТЫХ ТЕЛАХ

Аннотация—Представлен аналитический метод решения предложенной Лыковым системы линейных дифференциальных уравнений в частных производных при заданных начальных и граничных условиях. Уравнения Лыкова являются определяющими при анализе задач тепло- и массопереноса в капиллярно-пористых телах. Однако аналитический метод, позволяющий получить полные решения указанных уравнений, до сих пор не описан в литературе. Предложенный в данной статье метод решения иллюстрируется на примере нестационарной задачи для распределений температуры и влаги в деревянной пластине в процессе сушки. Полученные результаты сравиваются с опубликованными решениями, найденными с помощью метода конечных элементов, и экспериментальными данными для образцов ели. Описанный метод может быть применен к задачам тепло- и массопереноса в капиллярно-пористых телах.